Coordinate geometry (Grade XI)

Chapter : pair of st.lines

14.6 The pair of st.lines represented by second degree equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ are parallel to the pair of st. lines represented by homogeneous equation of second degree $ax^2+2hxy+by^2=0$.

Let $a_1x+b_1y+c_1=0$ ....(i) and $a_2x+b_2y+c_2=0$ ....(ii) are pair of st. lines given by $ax^2+2hxy+by^2+2gx+2fy+c=0$

Now combining equations (i) and (ii) ,

$(a_1x+b_1y+c_1)(a_2x+b_2y+c_2)=0$

Or, $a_1a_2x^2 + (a_1b_2+b_1a_2)xy+b_1b_2y^2+(a_1c_2+c_1a_2)x+(b_1c_2+c_1b_2)y+c_1c_2=0$ which is identical to equation $ax^2+2hxy+by^2+2gx+2fy+c=0$, so we get

$a_1a_2=a$, $a_1b_2+b_1a_2=2h$, $b_1b_2=b$, $a_1c_2+c_1a_2=2g$, $b_1c_2+c_1b_2=2f$ and $c_1c_2=c$

and the pair of st.lines which are parallel to lines (i) and (ii) and which passes through origin are given by

$a_1x+b_1y=0$ ....(iii) and $a_2x+b_2y=0$ ....(iv)
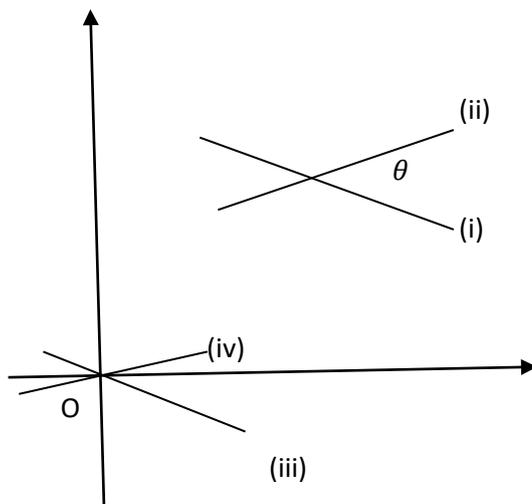
combining the above equations , we get

$(a_1x+b_1y)(a_2x+b_2y)=0$

Or, $a_1a_2x^2 + (a_1b_2+b_1a_2)xy+b_1b_2y^2=0$

Or, $ax^2+2hxy+by^2=0$ which is homogeneous equation of second degree.

Hence, proved.

NOTE: the angle between lines (i) and (ii) given by $ax^2+2hxy+by^2+2gx+2fy+c=0$ is equal to the angle between lines (iii) and (iv) given by $ax^2+2hxy+by^2=0$. And , if the angle be θ then it is calculated by the formula:

$$\theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2-ab}}{a+b}\right)$$

Example 1 : Find the angle between two lines represented by $2x^2+3xy+y^2+5x+2y-3=0$.

Solution : the angle between two lines represented by the above equation is the angle between $2x^2+3xy+y^2=0$.

So ,clearly a=2, b=1 and 2h=3(h=3/2) and if θ is the angle ,then

$\tan\theta=\pm\frac{2\sqrt{h^2-ab}}{a+b}=\pm\frac{2\sqrt{(3/2)^2-2.1}}{2+1}=\pm\frac{1}{3}$

Therefore , $\theta = \tan^{-1}\left(\pm\frac{1}{3}\right)$

Example 2: Find the equation to the st.lines passing through (1,1) and parallel to the lines represented by

$x^2-5xy+4y^2+x+2y-2=0$.

Solution :

The st.lines parallel to the lines represented by $x^2-5xy+4y^2+x+2y-2=0$ are parallel to lines represented by $x^2-5xy+4y^2=0$.

So , simplifying it we get

$x^2-4xy-xy+4y^2=0$

or, $x(x-4y)-y(x-4y)=0$

or, $(x-4y)(x-y)=0$

so either ,$x-4y=0$

or , $x-y=0$

And let the lines which are parallel to the above lines

Are $x-4y+k_1=0$ and $x-y+k_2=0$. From question , they pass through point (1,1)

Hence ,solving we get $k_1=3$ and $k_2=0$

Therefore the single required equation is $(x-4y+k_1)(x-y+k_2)=0$

$$Or, (x-4y+3)(x-y)=0$$

$$Or, x^2-5xy+4y^2+3x-3y=0$$